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DISTORTION OF ATOMIC-BEAM VELOCITY DISTRIBUTIONS DUE TO CLASSICAL HARD-SPHERE GAS SCATTERING

by Eugene J. Manista

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SUMMARY

The concept of the effective scattering cross section that arises primarily in studies of collisions among thermal-energy particles is reviewed. A formulation of the effective scattering cross section is developed from the definition of the collision cross section and is shown to reduce to the appropriate hard-sphere limit for the case of scattering in the center-of-mass system that is independent of relative velocity.

The distortion of the Maxwellian velocity distribution of an atomic beam due to its passage through a classical hard-sphere scattering gas is investigated. The effect of the scattering gas is reduced to a generalized hard-sphere scattering probability function $P(z)$ in which the collision radius d for a motionless scattering gas appears along with a function $\theta(z)$ that takes proper account of the motion of the scattering gas atoms. The function $P(z)$ gives the probability that a beam atom of velocity v undergoes no scattering in passing through a region containing a Maxwellian scattering gas. The parameter z is a reduced "velocity" that correlates the motion of the scattering gas and the incoming beam atom velocity. A table of the function $\theta(z)$ over the range $0 \leq z \leq 50$ is presented from which the distorted velocity distribution may be computed.

The effect of the scattering gas in shifting the entire vacuum beam distribution is also investigated. A first-order approximation for the shift in the peak velocity shows that the shift δ is to higher velocities and is linearly dependent on the collision cross section πd^2 and the density of scattering gas atoms present. These results are applied to the case of cesium-beam scattering by nitrogen, argon, and helium. The calculated shifts in the peak velocity are in good agreement with experimental results. The distorted velocity distribution calculated for the cesium-nitrogen interaction is also in good agreement with experimental results.

INTRODUCTION

The development of high-resolution velocity selectors (refs. 1 and 2) for thermal-energy particles allows observation of the velocity distribution present in an atomic beam. Moreover, distortion produced in the vacuum (unscattered) velocity distribution by the passage of an atomic beam through a scattering gas is directly observable along with any shift in the peak velocity of the vacuum distribution, or for that matter, any velocity shift of the entire distribution.

Previous investigators, such as Estermann et al. (ref. 3) in their gravity-deflection cesium-beam experiments and Miller and Kusch (ref. 1) in their potassium- and thallium-beam work with a helical velocity selector, observed deficiencies from the Maxwellian velocity distribution in the lower velocity groups as the source pressure was increased. They attributed the reduction in the number of atoms possessing the lower velocities to scattering effects associated with the cloud formation of beam atoms near the source slit.

The effect of hard-sphere elastic scattering in distorting the velocity distribution of an atomic beam has been discussed qualitatively by Rosin and Rabi (ref. 4). Their discussion, however, concerned itself primarily in showing that the distorted velocity distribution had a negligible effect in altering the value of the collision radius d determined from total atomic-beam-attenuation studies.

The present report extends the Rosin and Rabi qualitative discussion of the distortion produced in the velocity distribution of an atomic beam due to its passage through a Maxwellian scattering gas and gives quantitative results for estimating the distortion in the velocity distribution and the shift in the peak velocity of the unscattered beam.

SYMBOLS

All units are in the cgs system.

A	dimensionless quantity, $\ell \pi d^2 n_g$
B	parameter defined by eq. (21)
d	hard-sphere collision radius between beam particle and scattering gas particle
$f(v_g)$	scalar velocity distribution of scattering particles
$f(\bar{v}_g)$	vector velocity distribution of scattering particles
G	geometrical transmissivity of slotted-disk velocity selector
$G_n(x)$	general velocity distribution in terms of reduced quantities

$I(v)$	modified beam distribution after velocity selection
$I_0(v)$	velocity distribution present in effusing Maxwellian beam
$I_n(v)$	distribution function defined by eq. (21)
k	Boltzmann's constant
ℓ	path length of beam through scattering particles
m	mass of particle
n_g	density of scattering particles
$P(v)$	probability that incoming beam particle of velocity v is not scattered out of beam
$P(v,\theta)$	probability that incoming beam particle of velocity v is not scattered out of beam by more than angular resolution θ
$P_n(x)$	scattering probability function
$P(z)$	$P(v)$ in terms of reduced "velocities"
$Q(v,\theta)$	effective scattering cross section
T	kinetic temperature of particle distribution
v	scalar velocity of particle
$\overline{v}$	vector velocity of particle
v_{mn}	peak velocity of distribution $I_n(v)$
v_r	relative velocity
x	reduced "velocity" associated with incident-beam distribution
z	reduced "velocity" that correlates incoming-beam velocity with that of peak velocity of scattering-gas distribution
α	peak velocity of beam distribution in source
δ	amount of shift in peak velocity of vacuum distribution
θ	minimum detectable angle of scattering of incident beam particles in laboratory coordinate system
$\theta(z)$	function that takes proper account of velocity distribution present in scattering gas for hard-sphere collisions
θ', φ'	spherical coordinates in laboratory system
λ_0	classical mean free path for motionless hard-sphere gas
$\lambda(v)$	mean free path for velocity v
πd^2	actual hard-sphere cross section for motionless scattering gas
$\sigma(v_r)$	velocity dependent total cross section for scattering of two particles of relative velocity v_r

$d\sigma/d\omega$	differential cross section in center-of-mass system
$d\sigma/d\omega'$	differential cross section in laboratory system for scattering of beam particle of velocity v into solid angle specified by $\theta'\varphi'$
φ_g	azimuthal variable
$\psi(y)$	defined by eq. (9)
$d\omega/d\omega'$	ratio of solid angle differentials for center-of-mass and laboratory system

Subscripts:

a	incident-beam variables and constants
approx	approximately
g	scattering-gas variables and constants

Superscripts:

I	first approximation
II	second approximation
'	quantity after scattering has occurred

FORMULATION AND DISCUSSION

Effective Scattering Cross Section

Consider a collimated beam of monoenergetic particles of mass m and velocity $\overline{v}$ incident on a region of space containing a uniform density n_g of scattering gas particles. The scattering particles are assumed to have a normalized velocity distribution $f(\overline{v}_g)$, where g refers to scattering-gas variables and constants. The probability $P(v,\theta)$, where $v = |\overline{v}|$, that a beam particle of velocity $\overline{v}$ is not scattered out of the beam by more than an angle θ relative to the incident beam after traversing a distance ℓ of the scattering gas may be written in terms of experimentally measureable quantities as

$$P(v,\theta) = e^{-n_g \ell Q(v,\theta)} \tag{1}$$

where $Q(v,\theta)$ is the effective cross section for the scattering of the incident particles by an angle greater than the angular resolution θ. Experimentally, one measures $Q(v,\theta)$ by observing the change in the beam intensity and, hence, the change in $P(v,\theta)$ as the density-distance product $n_g\ell$ is varied. If the incoming beam velocity is large compared to the mean velocity of the scattering gas particles, $Q(v,\theta)$ approaches the true cross section for the interaction between the particles. In collisions involving thermal-energy particles, however, $Q(v,\theta)$ as expressed by equation (1) is an effective cross section for the interaction due to the motion of the scattering particles.

From the definition of the cross section as the total number of incident particles scattered by more than an angle θ per second divided by the incident beam intensity, the effective cross section $Q(v, \theta)$ may be written in general as

$$Q(v, \theta) = \frac{1}{v} \int_{\overline{v}_g} \int_{\substack{\omega' \\ \theta' > \theta}} \frac{d\sigma}{d\omega'} \left| \overline{v} - \overline{v}_g \right| f(\overline{v}_g) d\overline{v}_g \, d\omega' \tag{2}$$

In equation (2), $f(\overline{v}_g)$ is the fraction of scattering particles that have a velocity $\overline{v}_g$, $|\overline{v} - \overline{v}_g| = v_r$ is the magnitude of the relative velocity, and $d\sigma/d\omega'$ is the differential cross section in the laboratory coordinate system for the scattering of an incoming beam particle of velocity $\overline{v}$ and for the scattering gas particle of velocity $\overline{v}_g$ into the solid angle $d\omega'$ specified by the spherical angular coordinates θ' and φ'.

In general, the integrals required in equation (2) are often not obtainable in closed forms primarily because of the rather complicated relation existing between the dynamic variables that relate the differential cross section in the laboratory system to those in the center-of-mass system (ref. 5). The differential cross sections in the two coordinate systems are related according to

$$\frac{d\sigma}{d\omega'} = \frac{d\sigma}{d\omega} \frac{d\omega}{d\omega'} \tag{3}$$

where $d\sigma/d\omega$ is the differential cross section in the center-of-mass system and where $d\omega / d\omega'$ expresses the geometrical relation between the solid angles seen in both systems.

While equations (1) and (2) give the exact result in terms of the known angular resolution θ, considerable simplification results by allowing θ to take on the value zero. This procedure reduces the problem to that of the total cross section. Since the total cross section in the laboratory system has the same value as in the center-of-mass system, integration of equation (2) over all scattering angles yields

$$Q(v) = \frac{1}{v} \int_{\overline{v}_g} \sigma\left(\left|\overline{v} - \overline{v}_g\right|\right) \left|\overline{v} - \overline{v}_g\right| f(\overline{v}_g) d\overline{v}_g \tag{4}$$

where $\sigma(|\overline{v} - \overline{v}_g|)$ is now the velocity-dependent total cross section in the center-of-mass system and where $Q(v) \equiv Q(v, 0)$.

The relation that exists between $\overline{v}_r$, $\overline{v}$, and $\overline{v}_g$ is

$$v_r^2 = v^2 + v_g^2 - 2vv_g \cos \theta_g \tag{5}$$

where θ_g is the angle between the velocities $\overline{v}$ and $\overline{v}_g$ such that $\overline{v}_g$ is within the range of dv_g and $d\theta_g$; equation (5) may be used to write equation (4) as

$$Q(v) = \frac{2\pi}{v^2}\left[\int_0^v v_g f(v_g) dv_g \int_{v-v_g}^{v+v_g} \sigma(v_r) v_r^2 dv_r + \int_v^\infty v_g f(v_g) dv_g \int_{v_g-v}^{v_g+v} \sigma(v_r) v_r^2 dv_r\right] \tag{6}$$

In equation (6), $f(v_g)$ is the normalized scalar velocity distribution for the scattering gas particles (the scattering-gas velocity distribution $f(\overline{v}_g)$ has been assumed to be spherically symmetric in its phase space), and the integration over the azimuthal variable φ_g has been performed. The result expressed by equation (6) is as far as one is able to carry the calculation of $Q(v)$ without making a further assumption as to the explicit form of the total cross section $\sigma(v_r)$. Regardless of the form of the variation of $\sigma(v_r)$ with relative velocity, some care must be exercised in recognizing the proper limits when the integration over both v_r and v_g is performed. The first set of integrals in equation (6) is for the case where $v > v_g$, while the second set of integrals is for the case where $v < v_g$.

For the case of the hard-sphere collision approximation in which the total cross section is taken as independent of relative velocity, that is $\sigma(v_r) = \pi d^2$, equation (6) becomes

$$Q(v) = \frac{2\pi^2 d^2}{v^2}\left[\int_0^v v_g f(v_g) dv_g \int_{v-v_g}^{v+v_g} v_r^2 dv_r + \int_v^\infty v_g f(v_g) dv_g \int_{v_g-v}^{v_g+v} v_r^2 dv_r\right] \tag{7}$$

Classical Hard-Sphere Scattering Probability

The mean free path $\lambda(v)$ of an atom of mass m_a and velocity v, in a Maxwellian gas of mass m_g, temperature T_g, constant density n_g, and hard-sphere collision radius d, may be calculated either from kinetic theory methods (ref. 6) or by using equation (7) as

$$\lambda(v) = \frac{1}{n_g Q(v)} \tag{8a}$$

$$\lambda(v) = \frac{m_g}{2kT_g} \frac{v^2}{\sqrt{\pi}} \frac{1}{d^2 n_g} \frac{1}{\psi\left[v\left(\frac{m_g}{2kT_g}\right)^{1/2}\right]} \tag{8b}$$

where

$$\psi(y) = ye^{-y^2} + \frac{\sqrt{\pi}}{2}(1 + 2y^2)\mathrm{erf}(y) \tag{9}$$

and

$$\mathrm{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2}\, dt \tag{10}$$

The probability that the atom of velocity v will travel a distance ℓ through the gas without collision is

$$P(v) = e^{-\ell/\lambda(v)} \tag{11}$$

By defining the reduced "velocity" variable z as

$$z = v\left(\frac{m_g}{2kT_g}\right)^{1/2} \tag{12}$$

where z is a parameter that compares the incoming velocity of the beam atom to the peak velocity of the scattering-gas distribution, one may combine equations (8b), (11), and (12) to yield a probability function $P(z)$. The function $P(z)$ is dependent only on the collision radius d, the density of scatterers, and the function $\psi(z)/z^2$. The result is expressed by

$$P(z) = e^{-\ell\sqrt{\pi}\, d^2 n_g \left[\psi(z)/z^2\right]} \tag{13}$$

Equation (13) may be put into a more compact form by letting

$$A = \ell\pi d^2 n_g \tag{14}$$

If this definition of A is used, equation (13) takes the form of a generalized scattering probability function

$$P(z) = e^{-A\psi(z)/\sqrt{\pi}\, z^2} \tag{15}$$

The dimensionless quantity A is related to the classical mean free path obtained by considering the scattering gas as motionless. For a motionless scattering gas the mean free path λ_0 is independent of the velocity of the incoming particle and is given by

$$\lambda_0 = \frac{1}{\pi d^2 n_g} \tag{16}$$

The quantity A is the number of such classical mean free paths in the distance ℓ. The velocity dependence of the scattering gas and the velocity of the incoming particle are contained implicitly in the function $\theta(z)$:

$$\theta(z) = \frac{1}{\sqrt{\pi}} \frac{\psi(z)}{z^2} \tag{17}$$

By using the appropriate expansions for the erf(z), the following limits of $P(z)$ are obtained:

$$P(z) \rightarrow 0 \qquad \text{as} \qquad \exp\left(\frac{-2A}{\sqrt{\pi}\, z}\right) \qquad \text{as} \quad z \rightarrow 0 \tag{18a}$$

$$P(z) \rightarrow \exp(-A) \qquad \text{as} \quad z \rightarrow \infty \tag{18b}$$

The result expressed by equation (18a) is physically interpretable as meaning simply that as the velocity of the incoming particle is reduced, the distance between collisions goes to zero, and hence, the mean free path approaches zero. The result expressed by equation (18b) is consistent with the definition of the velocity independent mean free path λ_0.

A qualitative examination of the functional dependence of $P(z)$ on the incoming beam velocity shows that the lower velocities will be scattered more strongly than the higher velocities. The function $\theta(z)$ is expected to show its most rapid change in the range $0 \leq z \leq 1$.

Generalized Velocity Distribution for Vacuum Maxwellian Beams

The velocity distribution $I_0(v)$ present in a beam of atoms effusing from an isothermal enclosure through a narrow slit into a vacuum and satisfying the mean free path condition (ref. 7) is given by

$$I_0(v) = 2I_0 \frac{v^3}{\alpha^4} e^{-v^2/\alpha^2} \tag{19}$$

where

$$\alpha^2 = \frac{2kT_a}{m_a}$$

The distribution has been normalized to the total intensity I_0 (in units of atoms/(sec)(cm^2)). In equation (19) k is Boltzmann's constant, m_a is the mass of the atom, and T_a is the temperature of the enclosure in $^{\circ}$K.

In beam experiments, the atomic or molecular beam is often velocity analyzed or selected by a velocity selector of either the helical-slit design of Miller and Kusch (ref. 1) or the slotted-disk design of Hostettler and Bernstein (ref. 2). The effect of the velocity selector is to modify the distribution $I_0(v)$ to the following equation:

$$I(v) = 2I_0 \frac{Gv^4}{\alpha^4} e^{-v^2/\alpha^2} \tag{20}$$

The product Gv is the effective transmittance (refs. 1 and 2) of the selector, and G is the geometrical transmissivity.

It is well known that the form of the detected velocity-selected distribution is still further modified by the method of detecting the number of particles per second at the transmitted velocity. If a surface-ionization detector is employed, the distribution is unchanged from equation (20) except for some constant detection efficiency less than one; however, if an electron-bombardment ionizer is used to detect the incoming velocity-selected beam, the distribution given by equation (20) must be multiplied by a factor proportional to the inverse first power of v. The succeeding discussion presupposes that the beam is detected by a surface-ionization detector. The extension to the electron-bombardment ionizer detector with or without phase-sensitive detection is straightforward.

In order to make the results of the following discussion as generally applicable as possible, a distribution function $I_n(v)$ is defined as

$$I_n(v) = \frac{Bv^n}{\alpha^4} e^{-v^2/\alpha^2} \tag{21}$$

where

$B = 2I_0$ ($n = 3$; unselected beam)

$B = 2I_0G$ ($n = 4$; selected beam)

It is also convenient to define a reduced velocity $x = v/v_{mn}$, where v_{mn} is the peak velocity of the distribution $I_n(v)$ and is given by

$$v_{mn}^2 = \frac{n}{2}\alpha^2 \tag{22}$$

A generalized velocity distribution $G_n(x)$, corresponding to either an unselected ($n = 3$) or a selected ($n = 4$) beam, may be obtained from equation (21) in terms of x:

$$G_n(x) = B\left(\frac{n}{2}\right)^{n/2} \alpha^{n-4} x^n e^{-(n/2)x^2} \tag{23}$$

The distribution given by equation (23) has its maximum at $x = 1$ for n equal to 3 or 4. The distribution $G_n(x)$ will be useful in calculating the shift in the peak of the velocity distribution as the beam passes through a scattering gas and also will be useful in illustrating the shape of the unselected or selected vacuum-beam distribution.

Distorted Velocity Distribution

It is now possible to combine the results of the previous sections and obtain the distorted velocity distribution. The effect of the scattering gas in perturbing the vacuum distribution is taken as

$$I_n'(v) = I_n(v)P_n(v) \tag{24}$$

where the superscript prime is used to distinguish the distorted distribution. Equation (24) follows from considering the incident velocity distribution as made up of noninteracting, monoenergetic beams of velocity v and linearily combining the effect of the scattering gas on each such beam of the incident distribution. When the reduced velocity x is introduced, a convenient form of the distorted distribution $G_n'(x)$ is

$$G_n'(x) = B\left(\frac{n}{2}\right)^{n/2}\alpha^{n-4}x^n e^{-\frac{n}{2}x^2-\frac{A}{\sqrt{\pi}}\frac{2}{n}\left(\frac{m_aT_g}{m_gT_a}\right)\frac{\psi\left[x\left(\frac{n}{2}\frac{m_gT_a}{m_aT_g}\right)^{1/2}\right]}{x^2}} \tag{25}$$

where

$$P_n(x) = e^{-\frac{A}{\sqrt{\pi}}\frac{2}{n}\left(\frac{m_aT_g}{m_gT_a}\right)\frac{\psi\left[x\left(\frac{n}{2}\frac{m_gT_a}{m_aT_g}\right)^{1/2}\right]}{x^2}} \tag{26}$$

and the relation between the reduced velocities x and z is

$$x = z\left(\frac{2}{n}\frac{m_aT_g}{m_gT_a}\right)^{1/2} \tag{27}$$

Shift of Peak Velocity of Vacuum Beam by Hard-Sphere Scattering

The peak of the distorted velocity distribution (eq. (25)) is given by differentiating $G_n'(x)$ with respect to x and finding the value of x_{mn}' that makes

$$\left.\frac{dG_n'(x)}{dx}\right|_{x_{mn}'} = 0$$

The value of x_{mn}' is the solution to the following equation:

$$1 - x_{mn}'^2 + \frac{2A}{n^2}\left(\frac{m_aT_g}{m_gT_a}\right)\frac{\operatorname{erf}\left[x_{mn}'\left(\frac{n}{2}\,\frac{m_gT_a}{m_aT_g}\right)^{1/2}\right]}{x_{mn}'^2} = 0 \tag{28}$$

An approximate solution to equation (28) is obtained by considering the term involving the scattering as small compared to 1 and approximating the value of x_{mn}' in this term by 1. Under this approximation, $(x_{mn}')_{approx}$ to the first order is

$$(x_{mn}')_{approx} \cong 1 + \frac{1}{2}\,\frac{2n_g\ell\pi d^2}{n^2}\left(\frac{m_aT_g}{m_gT_a}\right)\operatorname{erf}\left[\left(\frac{n}{2}\,\frac{m_gT_a}{m_aT_g}\right)^{1/2}\right] \tag{29}$$

The effect of the scattering shifts the peak of the vacuum velocity distribution toward the higher velocities by an amount that is linearily dependent on the effective hard-sphere collision cross section πd^2. The amount of the shift δ_n^I, where $(x_{mn}')_{approx} = 1 + \delta_n^I$, is also linearily dependent on the density of scattering gas atoms present.

A better approximate solution to equation (28) is obtained by replacing x_{mn}' by $1 + \delta_n^{II}$, expanding the error function about $x_{mn}' = 1$, and keeping only first-order terms in δ_n^{II}. The result of this approximation scheme is

$$\delta_n^{II} = \frac{\delta_n^I}{1 + 2\delta_n^I\left[1 - \frac{1}{\sqrt{\pi}}\left(\frac{n}{2}\frac{m_gT_a}{m_aT_g}\right)^{1/2}\frac{e^{-\frac{n}{2}\frac{m_gT_a}{m_aT_g}}}{\operatorname{erf}\left[\left(\frac{n}{2}\frac{m_gT_a}{m_aT_g}\right)^{1/2}\right]}\right]} \tag{30}$$

A comparison of approximations I and II is given in table I. The calculated shifts are for a beam of cesium atoms at a source temperature of 450° K that has passed through various scattering gases at a temperature of 300° K such that the vacuum peak velocity is attenuated by one-half. The shifts presented in table I are calculated for $n = 4$ in order to make the results directly applicable to velocity-selector data.

TABLE I. - SHIFTS IN VACUUM PEAK VELOCITY OF CESIUM BEAM THAT HAS BEEN VELOCITY SELECTED AFTER PASSING THROUGH VARIOUS SCATTERING GASES

[Beam temperature, 450° K; scattering-gas temperature, 300° K; attenuation of vacuum peak, 0.5.]

Scattering gas	First approximation of amount of shift in peak velocity of vacuum distribution, δ_4^I (a)	Second approximation of amount of shift in peak velocity of vacuum distribution, δ_4^{II} (a)	Amount of shift in peak velocity of vacuum distribution, δ_4 (b)
Hydrogen	0.083	0.077	-----
Helium	.082	.075	0.084
Neon	.066	.061	-----
Nitrogen	.060	.055	.053
Argon	.052	.048	.053
Krypton	.035	.033	-----
Xenon	.025	.024	-----

[a]Theoretical.

[b]Observed (ref. 8).

As can be seen from either the inspection of equation (30) or the values in table I, the calculated shift is well estimated by δ_n^I for scattering gases comparable in mass with the incoming beam atom and for attenuations that are not too large. The former is due, in part, to the relative insensitiveness of the value of the erf(y) for small changes in the value of

$$y = \left[\left(\frac{n}{2}\right)\left(\frac{m_aT_g}{m_gT_a}\right)\right]^{1/2}$$

for these cases; however, for large attenuations the approximation scheme used to solve equation (28) obviously does not hold. The shift is largest for the cases of the lighter mass scattering atoms since the variation in the value of $P_n(x)$ (eq. (26)) causes appreciable distortion in the vacuum velocity distribution over the entire velocity

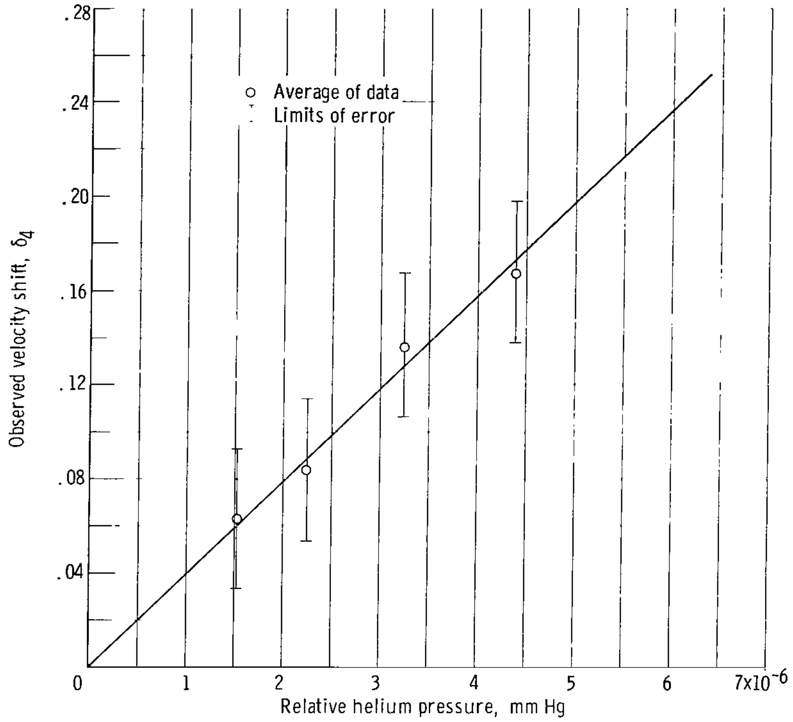

Figure 1. - Observed velocity shift in peak velocity of cesium atomic beam as function of relative helium scattering gas pressure. Data are from reference 8 and correspond to cesium temperature of 450° K and helium temperature of 300° K.

range of the original distribution.

In a recent cesium-beam experiment (ref. 8) performed with a high-resolution slotted-disk velocity selector and with nitrogen, argon, and helium as the scattering gases, observable shifts in the vacuum distribution were detected and measured. The experimental conditions were similar to those specified by the conditions under which the values of the shift were computed in table I. The observed shifts are within ±10 percent of the calculated values. Figure 1 is a plot of the observed shifts in the peak velocity of the vacuum cesium beam for the case of scattering by helium (ref. 8). The abscissa in figure 1 is the relative helium pressure as indicated by an ion gage. The data confirm the linear relation between the shift in the peak velocity (eq. (29)) and the density of the helium atoms present. The error bars reflect the experimental difficulty involved in determining the peak velocity of the distribution.

Computation of Distorted Velocity Distribution for Cesium Beam in Nitrogen

The function $\theta(z)$ defined in equation (17) has been evaluated over the range $0 \leq z \leq 50$. The choice of the range of z allows the distorted velocity distribution to be

TABLE II. - VALUES OF FUNCTION THAT TAKES PROPER ACCOUNT OF VELOCITY DISTRIBUTION PRESENT IN SCATTERING GAS FOR HARD-SPHERE COLLISIONS

Reduced velocity, z	Function of reduced velocity, $\theta(z)$	Reduced velocity, z	Function of reduced velocity, $\theta(z)$	Reduced velocity, z	Function of reduced velocity, $\theta(z)$	Reduced velocity, z	Function of reduced velocity, $\theta(z)$
0.01	112.8418	0.39	3.037788	0.77	1.739217	1.75	1.162839
.02	56.42654	.40	2.969048	.78	1.723600	1.80	1.154003
.03	37.62396	.41	2.903827	.79	1.708445	1.85	1.145856
.04	28.22455	.42	2.841875	.80	1.693734	1.90	1.138329
.05	22.58641	.43	2.782961	.81	1.679451	1.95	1.131363
.06	18.82890	.44	2.726878	.82	1.665579	2.00	1.124904
.07	16.14604	.45	2.673438	.83	1.652103	2.20	1.103278
.08	14.13483	.46	2.622465	.84	1.639008	2.40	1.086798
.09	12.57138	.47	2.573803	.85	1.626279	2.60	1.073963
.10	11.32138	.48	2.527306	.86	1.613905	2.80	1.063775
.11	10.29933	.49	2.482840	.87	1.601872	3.00	1.055555
.12	9.448240	.50	2.440284	.88	1.590167	3.20	1.048828
.13	8.728663	.51	2.399524	.89	1.578780	3.40	1.043253
.14	8.112414	.52	2.360456	.90	1.567699	3.60	1.038580
.15	7.578828	.53	2.322983	.91	1.556913	3.80	1.034626
.16	7.112404	.54	2.287016	.92	1.546413	4.00	1.031250
.17	6.701289	.55	2.252472	.93	1.536189	4.50	1.024691
.18	6.336264	.56	2.219273	.94	1.526231	5.00	1.020000
.19	6.010051	.57	2.187350	.95	1.516531	5.50	1.016529
.20	5.716828	.58	2.156634	.96	1.507079	6.00	1.013889
.21	5.451880	.59	2.127065	.97	1.497869	6.50	1.011834
.22	5.211352	.60	2.098583	.98	1.488891	7.00	1.010204
.23	4.992056	.61	2.071135	.99	1.480139	7.50	1.008889
.24	4.791339	.62	2.044669	1.00	1.471605	8.00	1.007813
.25	4.606970	.63	2.019139	1.05	1.431977	8.50	1.006920
.26	4.437062	.64	1.994501	1.10	1.396872	9.00	1.006173
.27	4.280008	.65	1.970711	1.15	1.365652	9.50	1.005540
.28	4.134429	.66	1.947731	1.20	1.337789	10.00	1.005000
.29	3.999137	.67	1.925524	1.25	1.312837	15.00	1.002222
.30	3.873103	.68	1.904056	1.30	1.290422	20.00	1.001250
.31	3.755430	.69	1.883293	1.35	1.270226	25.00	1.000800
.32	3.645334	.70	1.863204	1.40	1.251980	30.00	1.000556
.33	3.542125	.71	1.843761	1.45	1.235451	35.00	1.000408
.34	3.445194	.72	1.824936	1.50	1.220439	40.00	1.000312
.35	3.354003	.73	1.806703	1.55	1.206772	45.00	1.000247
.36	3.268072	.74	1.789037	1.60	1.194300	50.00	1.000200
.37	3.186975	.75	1.771916	1.65	1.182895		
.38	3.110328	.76	1.755316	1.70	1.172441		

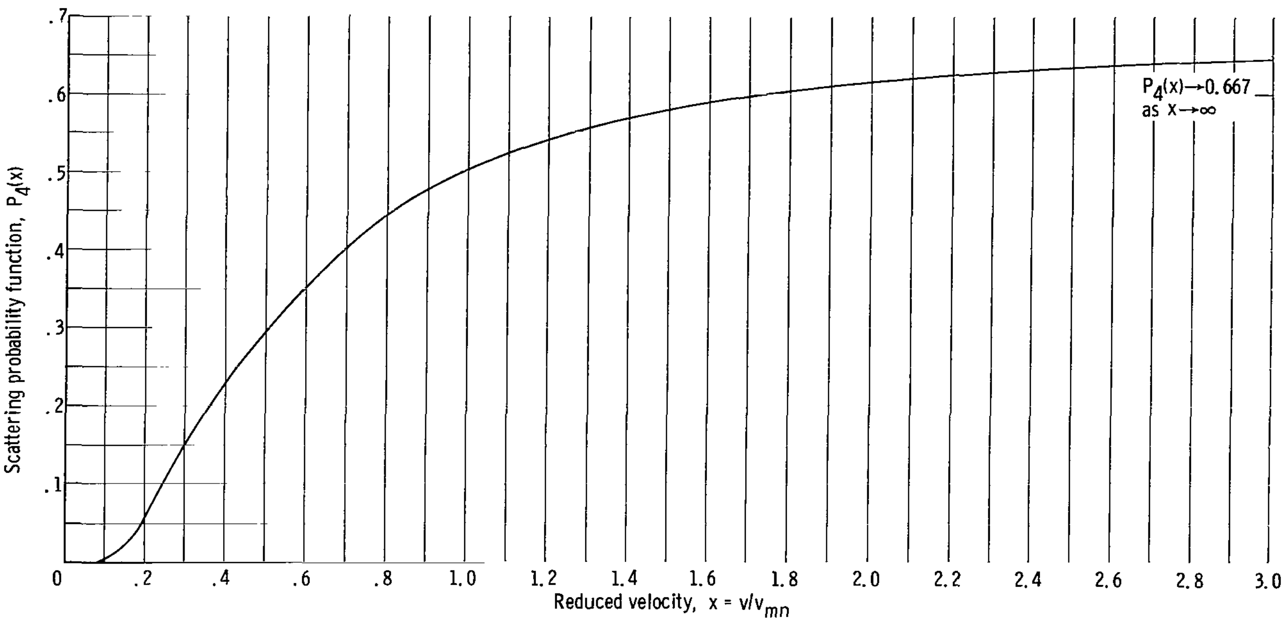

Figure 2. - Calculated velocity-selected scattering probability function as function of reduced velocity. Function $P_4(x)$ calculated for cesium beam at cesium temperature of 450° K and nitrogen scattering gas at nitrogen temperature of 300° K. Value of A chosen to make $P_4(1) = 0.5$.

calculated for a wide variation of beam atoms and scattering gas atoms in the mass range of hydrogen to mercury. The evaluation of $\theta(z)$ was performed on an IBM 7090 digital computer. Since the strongest variation in $\theta(z)$ is over the range $0 \leq z \leq 1$, the values of $\theta(z)$ were computed in 0.01 steps. The choice of the steps in z for $z > 1$ were dictated mainly by the change in $\theta(z)$ for the value of the argument. Table II presents the results of the evaluation of $\theta(z)$.

The distorted velocity distribution is calculated from the values of $\theta(z)$ by the following method. The numerical factor that relates the value of x and z is computed from equation (27) for the specific case under consideration along with the value of the constant A. The probability $P_n(x)$ that a particular value of x will pass unscattered through the scattering gas is given by evaluating equation (26) for that value of z corresponding to the value of x. The values of $\theta(z)$ for z values not tabulated may be obtained by interpolation of the values given in table II, or else the procedure may be reversed to that of calculating the value of x corresponding to the value of z listed in the table. The distorted distribution is then simply obtained by multiplying the value of $G_n(x)$ by the value of the determined $P_n(x)$.

A calculation of the distortion expected in a velocity-selected cesium beam that is attenuated to one-half of its value at the vacuum peak velocity by nitrogen-gas scattering is used to illustrate the method. A beam temperature of 450° K and a scattering gas temperature of 300° K are chosen. The numerical factor that relates x and z for this case is $x = 1.258z$. The quantity A has the value 0.407. Figure 2 is a plot of the cal-

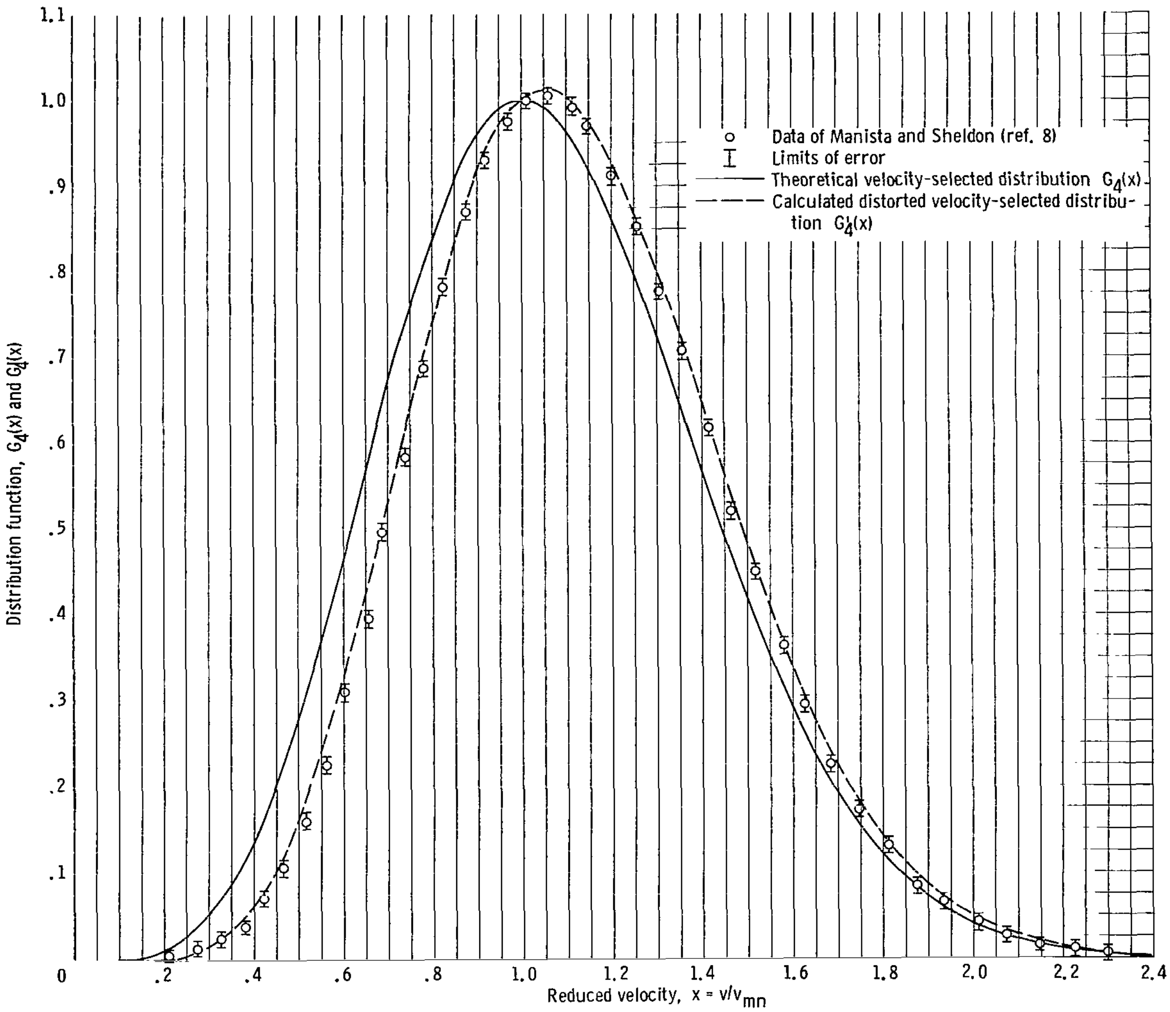

Figure 3. - Observed normalized velocity-selected distribution function present in cesium beam after passing through scattering gas of nitrogen as function of reduced velocity. Data are from reference 8 and correspond to cesium temperature of 450° K and nitrogen temperature of 300° K. Velocity at peak ($x_m = 1$) attenuated by 0.5 from that of vacuum condition.

culated $P_4(x)$ against the reduced velocity x. As expected, the slower moving atoms in the beam ($x < 1$) are the ones most strongly attenuated or scattered out of the original vacuum distribution.

In order to compare the calculated distorted distribution with the results of a recent cesium-beam experiment (ref. 8) that involved nitrogen-gas scattering, the theoretical distribution $G_n(x)$ (eq. (23)) was normalized to 1 at $x = 1$. The results are shown in figure 3. The solid curve is the theoretical velocity-selected vacuum distribution $G_4(x)$. The dashed curve is the calculated distorted velocity-selected distribution $G_4'(x)$ normalized to 1 at $x = 1$, and the points shown are the data obtained in the cesium-nitrogen experiments of Manista and Sheldon (ref. 8). The agreement between the observed distribution and the calculated one is excellent for this case.

Notice that the entire distribution $G_4(x)$ is shifted toward higher absolute velocities after passing through the scattering gas. On the surface it appears that the scattering process has violated the law of energy conservation in so far as the velocity distribution in the beam after scattering now corresponds to a higher mean energy. The problem resolves itself when one realizes that the particles under observation, $G_4'(x)$, are those from the original distribution that have not been scattered out of the beam; if one were able to observe the particles that have been scattered out of the beam, one would find this scattered distribution composed of particles that are shifted toward lower absolute velocities and, hence, to a lower mean energy.

A final word in regard to the interpretation of velocity-distribution experiments is appropriate. If the observed peak velocity corresponding to a maximum signal to the detector is chosen as the normalizing point in order to compare the experimental distribution with that of the Maxwellian distribution, one may inadvertently, under conditions of appreciable cloud scattering at the beam-source slit, distort the experimental distribution from the true distribution. Normalizing to the observed peak velocity in these cases has the effect of introducing apparent deficiencies in the number of higher velocity particles in the distribution and, also, an underestimation of the deficiencies in the number of lower velocity particles.

CONCLUDING REMARKS

A generalized formulation of the effective scattering cross section for particles whose velocities are comparable was presented. The formulation was shown to reduce to that of the classical kinetic-theory velocity-dependent mean free path in the hard-sphere approximation.

The scattering probability $P(v)$ for a beam atom of velocity v in a Maxwellian scattering gas was investigated. The scattering probability was written in terms of a universal function $\theta(z)$, where z was a parameter that took into account the relative motion of the beam atom and the scattering gas atoms. A table of the computed values of $\theta(z)$ was given for the range $0 \leq z \leq 50$.

A method of computing the distortion of a Maxwellian velocity distribution of an atomic beam due to its passage through a scattering gas was outlined. The calculation was applied to the case of cesium-beam scattering by nitrogen, and the results were shown to be in excellent agreement with experiment.

An analytical study of the shift in the peak velocity of a Maxwellian distribution as the distribution passed through a classical hard-sphere gas was made. An approximate expression for the shift in the peak velocity of the distribution was derived. The results of the calculation showed that the velocity shift was always toward the higher velocities in the distribution and was strongly dependent on the amount of attenuation or scattering that had occurred. The calculated shifts for the specific case of cesium-beam scattering by nitrogen, argon, and helium were shown to be in good agreement with the recent experiments of Manista and Sheldon (ref. 8).

The hard-sphere approximation of classical kinetic theory is expected to be a valid approach to the method of calculating the distortion of the beam distribution for the cases in which the mean velocity of the scattering-gas distribution is large compared to the mean beam velocity. This implies that in equation (6) the actual relative velocity variation of the total cross section is over a limited range ($\sigma(v_r) \propto v_r^{-2/5}$ due to the presence of a van der Waal type inverse sixth-power attraction between the neutral atoms (ref. 9)) and that the cross section may be replaced by an effective hard-sphere value without too much influence on the resulting integrations over v_r and v_g.

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